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Unique Paper Code : 2351302

Name of the Paper : Analysis-II (Real Functions)

Name of the Course : Maths(H) – II

Semester : III

Duration : 3 Hours Maximum Marks : 75

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*All questions are compulsory.*

*Attempt any **three** parts from each question.*

1. a. Let $A \subseteq R$ and $c \in R$ be a cluster point of $A$ and $f: A \longrightarrow R$, then define limit of function $f$ at $c$. Also show that if $f$ has a limit at $c \in R$, then $f$ is bounded on some neighbourhood of $c$. 5

   b. Let $c \in R$. Use $\varepsilon$-$\delta$ definition to show that $\lim_{x \longrightarrow c} x^3 = c^3$. 5

   c. State Sequential Criterion of Limits. Using sequential criterion, prove that $\lim_{x \to 0} \frac{1}{x^2}$, $x > 0$ does not exist. 5

   d. Let $A \subseteq R$, let $f, g, h: A \longrightarrow R$ and let $c \in R$ be a cluster point of $A$. If $f(x) \leq g(x) \leq h(x)$ for all $x \in A, x \neq c$, and if $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$, then show that $\lim_{x \to c} g(x) = L$. 5

2. a. Let $A \subseteq R$, let $f$ and $g$ be functions from $A$ to $R$ and let $c \in R$ be a cluster point of $A$. Suppose that $f$ is bounded in a neighbourhood of $c$ and $\lim_{x \to c} g(x) = 0$. Prove that $\lim_{x \to c} (fg)(x) = 0$. 5

   b. Let $f(x) = \frac{1}{(e^{1/x} + 1)}$ $for\ x \neq 0$, then find $\lim_{x \to 0-} f(x)$ and $\lim_{x \to 0+} f(x)$. 5

   c. Determine the points of continuity of the function $f(x) = [\![x]\!], x \in R$, where $[\![x]\!]$ denotes the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$. 5

**d.** Let $g: R \to R$ be defined by

$$g(x) = \begin{cases} x, & \text{for } x \text{ rational} \\ 0, & \text{for } x \text{ irrational} \end{cases}$$

Find all the points at which g is continuous. **5**

**3. a.** Let $A \subseteq R$ and $f: A \to R$ such that $f(x) \geq 0$ for all $x \in A$. Show that if $f$ is continuous at $c \in A$, then $\sqrt{f}$ is continuous at $c$. **5**

**b.** Let $A \subseteq R$. Let $f: A \to R$ and $g: A \to R$ be continuous on $A$. Show that $f + g$ is continuous on $A$. **5**

**c.** Let $f: R \to R$ be continuous on $R$ and let $P = \{x \in R: f(x) > 0\}$. If $c \in P$ show that there exists a neighbourhood $V_\delta(c) \subseteq P$. **5**

**d.** Suppose that $f$ is a real valued continuous function on $R$ and that $f(a)f(b) < 0$ for some $a, b \in R$. Prove that there exists $x$ between $a$ and $b$ such that $f(x) = 0$. Prove that $x = \cos x$ for some $x$ in $(0, \frac{\pi}{2})$. **5**

**4. a.** Show that every uniformly continuous function on $A \subseteq R$ is continuous on $A$. Is the converse true? Justify your answer. **5**

**b.** Show that the function $\sin(1/x), x \neq 0$ is not uniformly continuous on $(0, \infty)$. **5**

sin x

**c.** Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that $f$ is differentiable at $x = 0$ and find $f'(0)$. **5**

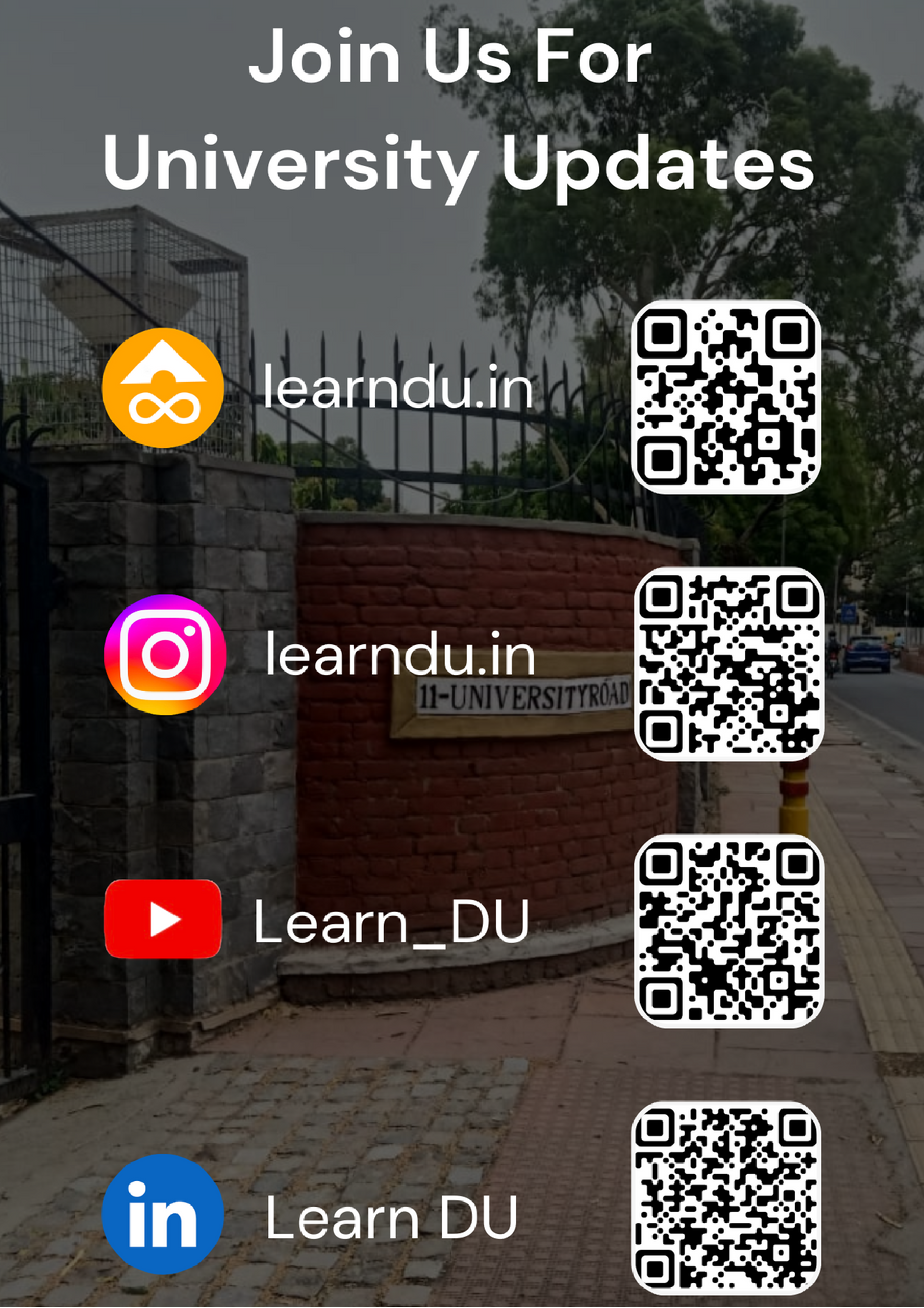
**d.** Let $f: I \to R$ is differentiable on the interval $I$. Prove that $f$ is increasing on $I$ if and only if $f'(x) \geq 0$ for all $x \in I$. **5**

**5. a.** Use the Mean Value theorem to prove $(x-1)/x < \ln x < x - 1$ for $x > 1$. **5**

**b.** For the function $f: R \to R$ given by $f(x) = 3x - 4x^2$, find the points of relative extrema. Also find the intervals on which the function is increasing and those on which it is decreasing. **5**

**c.** State and prove Cauchy's Mean Value theorem. **5**

**d.** Obtain Maclaurin's series expansion for the function $f(x) = \cos x, x \in R$ **5**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