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Sr. No. of Question Paper : 6623

HC

Unique Paper Code : 32351301

Name of the Paper : Theory of Real Functions

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top of this question paper.
2. All questions are compulsory.
3. Attempt any **three** parts from

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1. (a) Use the ϵ - δ definition of the limit to find $\lim_{x \rightarrow 2} f(x)$

where $f(x) = \frac{1}{1-x}$. (5)

- (b) State and prove Sequential Criterion for Limits. (5)

- (c) State Squeeze Theorem. For $n \in \mathbb{N}$, $n \geq 3$, derive the inequality, $-x^2 \leq x^n \leq x^2$ for $-1 < x < 1$. Hence prove

that $\lim_{x \rightarrow 0} x^n = 0$ for $n \geq 3$, assuming that $\lim_{x \rightarrow 0} x^2 = 0$. (5)

P.T.O.



- (d) Let f, g be defined on $A \subseteq \mathbb{R}$ to \mathbb{R} , and let c be a cluster point of A . Suppose that f is bounded on a neighbourhood of c and that $\lim_{x \rightarrow c} g = 0$. Prove that
- $$\lim_{x \rightarrow c} fg = 0. \quad (5)$$

2. (a) Let $c \in \mathbb{R}$ and let f be defined for $x \in (c, \infty)$ and $f(x) > 0$ for all $x \in (c, \infty)$. Show that $\lim_{x \rightarrow c} f = \infty$ if and only if $\lim_{x \rightarrow c} \frac{1}{f} = 0$.
- $$(5)$$

(b) Prove that

$$(i) \quad \lim_{x \rightarrow 0} \frac{1}{\sqrt{|x|}} = +\infty, \quad x \neq 0$$

$$(ii) \quad \lim_{x \rightarrow 0^-} e^{1/x} = 0, \quad x \neq 0. \quad (5)$$

- (c) Let $A = \mathbb{R}$ and let f be Dirichlet's function defined by

$$g(x) = \begin{cases} 1, & \text{for } x \text{ rational} \\ -1, & \text{for } x \text{ irrational} \end{cases}$$

Show that f is discontinuous at any point of \mathbb{R} . (5)

- (d) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be continuous at c and let $f(c) > 0$. Show that there exists a neighbourhood $V_\delta(c)$ of c such that if $x \in V_\delta(c)$ then $f(x) > 0$.
- $$(5)$$

3. (a) Determine the points of continuity of the function $f(x) = x - \llbracket x \rrbracket$, $x \in \mathbb{R}$, where $\llbracket x \rrbracket$ denotes the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$. (5)
- (b) Let $A, B \subseteq \mathbb{R}$, let $f: A \rightarrow \mathbb{R}$ be continuous on A , and let $g: B \rightarrow \mathbb{R}$ be continuous on B . If $f(A) \subseteq B$, show that the composite function $g \circ f: A \rightarrow \mathbb{R}$ is continuous on A . (5)
- (c) Let f be a continuous real valued function defined on $[a, b]$. Show that f is a bounded function. (5)
- (d) Prove that a polynomial of odd degree has at least one real root. (5)
4. (a) Define uniform continuity of a function on a set $A \subseteq \mathbb{R}$. Show that every uniformly continuous function on A is continuous on A . Is the converse true? Justify your answer. (5)
- (b) Show that the function \sqrt{x} is uniformly continuous on $[0, \infty)$. (5)
- (c) Let I, J be intervals in \mathbb{R} , let $g: I \rightarrow \mathbb{R}$ and $f: J \rightarrow \mathbb{R}$ be functions such that $f(J) \subseteq I$ and let $c \in J$. If f is differentiable at c and if g is differentiable at $f(c)$, show that the composite function $g \circ f$ is differentiable at c and $(g \circ f)'(c) = g'(f(c)) \cdot f'(c)$. (5)



(d) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is differentiable at $x = 0$ and find $f'(0)$. (5)

5. (a) Let f be continuous on $[a, b]$ and differentiable on (a, b) . Prove that f is increasing on $[a, b]$ if and only if $f'(x) \geq 0 \quad \forall x \in [a, b]$. (5)

(b) State Darboux's Theorem. Suppose that if $f: [0, 2] \rightarrow \mathbb{R}$ is continuous on $[0, 2]$ and differentiable on $(0, 2)$, and that $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that there exists $c_1 \in (0, 1)$ such that $f'(c_1) = 1$

(ii) Show that there exists $c_2 \in (1, 2)$ such that $f'(c_2) = 0$ n

(iii) Show that there exists $c \in (0, 2)$ such that $f'(c) = 1/3$. (5)

(c) Find the Taylor series for $\cos x$ and indicate why it converges to $\cos x \quad \forall x \in \mathbb{R}$. (5)

(d) Define a convex function on $[a, b]$. Check the convexity of the following functions on given intervals :

(i) $f(x) = x - \sin x$, $x \in [0, \pi]$.

(ii) $g(x) = x^3 + 2x$, $x \in [-1, 1]$. (5)

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