Roll No. ☐☐☐☐☐☐☐☐☐☐

S. No. of Question Paper : **6625**

Unique Paper Code : **32351303**　　　　　　**HC**

Name of the Paper : **Multivariate Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 Hours**　　　　　　Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

Attempt any *five* questions from each Section.

All questions carry equal marks.

Section I

1. Let *f* be the function defined by :

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$$

Is *f* continuous at (0, 0) ? Explain.

2. Find the equation for each horizontal tangent plane to the surface :

$$z = 5 - x^2 - y^2 + 4y.$$

3. Let f and g be twice differentiable functions of one variable and let $u(x, t) = f(x + ct) + g(x - ct)$ for a constant c. Show that :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

4. Let f have continuous partial derivatives and suppose the maximal directional derivative of f at $P_0(1, 2)$ has magnitude 50 and is attained in the direction from P_0 towards $Q(3, -4)$. Use this information to find $\nabla f(1, 2)$.

5. Find the absolute extrema of $f(x, y) = x^2 + xy + y^2$ on the closed bounded set S where S is the disk $x^2 + y^2 \le 1$.

6. Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

Section II

7. Find the area of the region D by setting double integral, where D is bounded by the parabola $y = x^2 - 2$ and the line $y = x$.

8. Write an equivalent integral with the order of integration reversed and then compute the integral :

$$\int_0^4 \int_0^{4-x} xy \, dy \, dx.$$

9. Calculate the Jacobian of transformation from rectangular to polar coordinates and hence evaluate the integral :

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}} \, dx \, dy.$$

10. Find the volume V of the solid bounded above by the cylinder $y^2 + z = 4$ and below by $x^2 + 3y^2 = z$.

11. Evaluate the integral below, where D is the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$:

$$\iiint_D z \, dx \, dy \, dz.$$

12. Let D be the region in the xy-plane that is bounded by the co-ordinate axes and the line $x + y = 1$. Use the suitable change of variable to compute the integral :

$$\iint\limits_{D} \left(\frac{x - y}{x + y} \right)^6 dy\, dx.$$

Section III

13. State Green's theorem for simply connected regions. Use Green's theorem to find the work done by the force field $\mathbf{F}(x, y) = (e^x - y^3)\mathbf{i} + (\cos y + x^3)\mathbf{j}$ along the circle $x^2 + y^2 = 1$ in anticlockwise direction.

14. Give the geometrical interpretation of the surface integral $\iint ds$ over piecewise smooth surface S. Evaluate the surface integral $\iint xz\, ds$ over the surface S which is the part of the plane $x + y + z = 1$ that lies in the first octant.

15. Verify Stokes' theorem for the vector field $\mathbf{F}(x, y, z) = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$ taking surface σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

16. State and prove Divergence theorem.

17. Verify that the vector field $\mathbf{F}(x, y) = (e^x \sin y - y)\mathbf{i} + (e^x \cos y - x - 2)\mathbf{j}$ is conservative using cross partial test. Use a line integral to find the area enclosed by the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

18. Let E be the solid unit cube with opposing corners at the origin and $(1, 1, 1)$ with faces parallel to co-ordinate planes. Let S be the boundary surface of E oriented with the outward pointing normal. If $\mathbf{F}(x, y, z) = 2xy\mathbf{i} + 3ye^z\mathbf{j} + x \sin z\, \mathbf{k}$, find the integral

$$\iint \mathbf{F.n}\, ds$$ over surface S using divergence theorem.

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