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Sl. No. of Ques. Paper: 5711

Unique Paper Code : 235304

Name of Paper : Algebra – II (MAHT-303)

Name of Course : B.Sc. (Hons.) Mathematics

Semester : III

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immedi
on receipt of this question paper.)

Do any two parts from each questions.

Questions

1. (a) Let $G = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in \mathbb{R}, a \neq 0 \right\}$. Show that G is a group under matrix multiplication. (6)

   (b) (i) Prove that if G is a group with the property that square of every element is identity then G is abelian.

   (ii) Define center of a group G. Show that center of a group G is an abelian subgroup of G. (2 + 4)

   (c) Define order of an element. Consider the element $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. What is the order of A in (i) $SL(2, \mathbb{R})$ (ii) $SL(2, \mathbb{Z}_p)$, p is a prime. (6)

2. (a) Let $G = \langle a \rangle$ be a cyclic group of order n. Prove that $G = \langle a^k \rangle$ if and only if $\gcd(n, k) = 1$. Find all the generators of Z_{20}. (6.5)

   (b) Suppose that a and b are group elements that commute have orders m and n respectively. If $\langle a \rangle \cap \langle b \rangle = \{e\}$. Prove that the group contains an element whose order is the least common multiple of m and n. Show that this need not be true if a and b do not commute. (6.5)

(c) Let 'a' be a fixed element of a group G. Define centralizer of the element a. Show that $Z(G) = \bigcap_{a \in G} C(a)$. **(6.5)**

3. (a) (i) Prove that product of two odd permutation is an even permutation.

(ii) Show that $Z(S_n) = \{\varepsilon\}$ for $n \geq 3$. **(2 + 4)**

(b) Show that if H is a subgroup of S_n then every member of H is an even permutation or exactly half of them are even. **(6)**

(c) (i) Let H and K be subgroups of a group G. If $|H| = 12$ and $|K| = 35$, find $|H \cap K|$.

(ii) Find all left cosets of $\{1, 11\}$ in $U(30)$. **(2 + 4)**

4. (a) State and prove Lagrange's theorem for finite groups. **(6.5)**

(b) (i) Prove that every subgroup of D_n of odd order is cyclic.

(ii) Prove or disprove $Z \times Z$ is a cyclic group. **(3.5 + 3)**

(c) Define index of a subgroup in a group. Show that Q, the group of rational numbers under addition has no proper subgroup of finite index. **(6.5)**

5. (a) Let G be a group and H a normal subgroup of G. The set $G/H = \{aH \mid a \in G\}$ is a group under the operation $(aH)(bH) = abH$. **(6)**

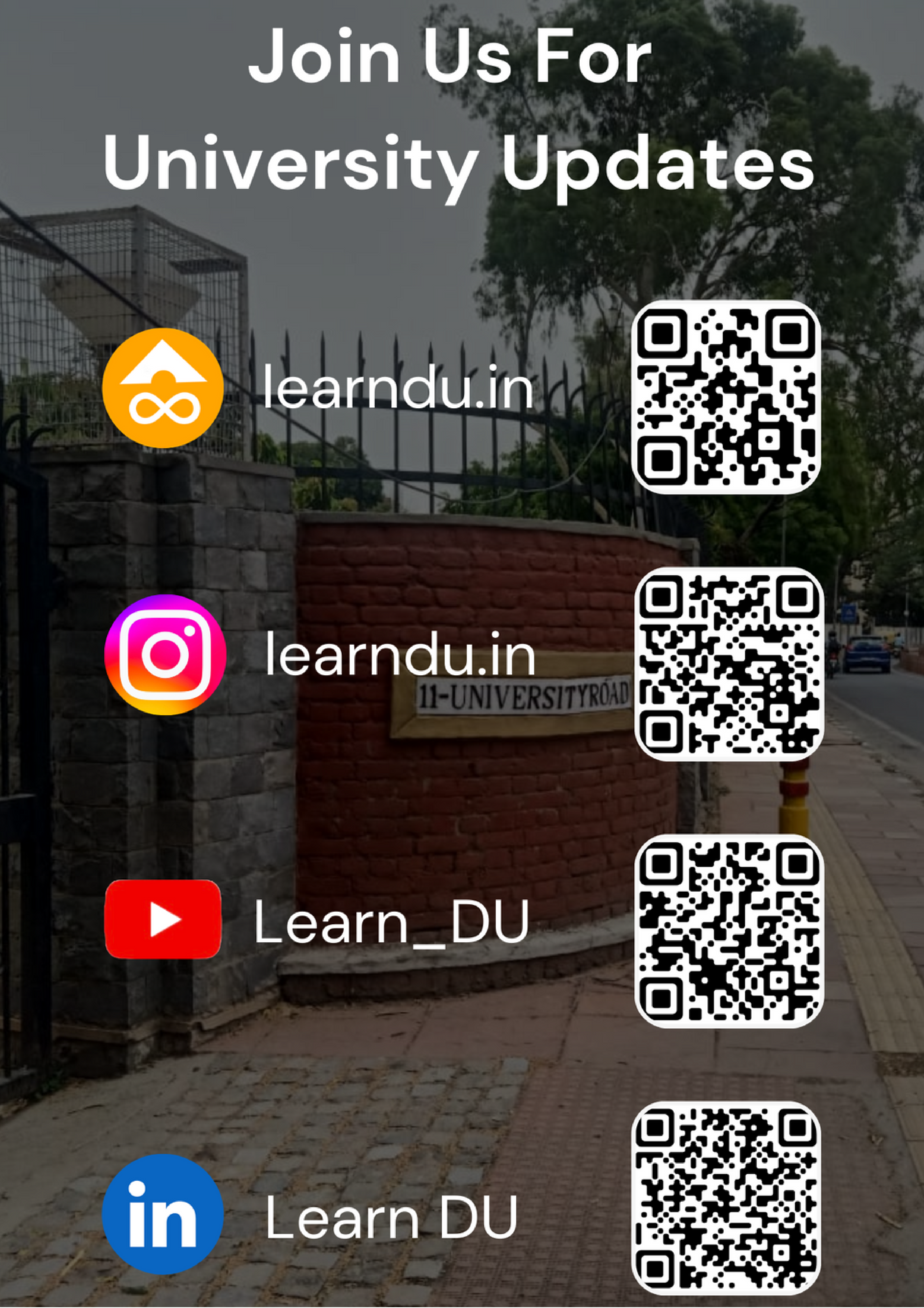
(b) Let N be a normal subgroup of a finite group G. If N is cyclic, prove that every subgroup of N is normal in G. **(6)**

(c) Determine all the homomorphisms from Z_{12} to Z_{30}. **(6)**

6. (a) Suppose that φ is an isomorphism from a group G onto a group G^*. Prove that G is cyclic if and only if G^* is cyclic. Hence show that Z, the group of integers under addition is not isomorphic to Q, the group of rationals under addition. **(6.5)**

(b) State and prove Cayley's theorem. **(6.5)**

(c) Let M and N be normal subgroups of a group G and $N \subseteq M$. Prove that $(G/N)/(M/N) \cong G/M$. **(6.5)**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