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S. No. of Question Paper : 90

(12)

Unique Paper Code : 32351301

Name of the Paper : Theory of Real Functions

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : III

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts from each question

All questions are compulsory.

1. (a) Let $A \subseteq \mathbb{R}$, $f: A \rightarrow \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A . Then prove that f can have only one limit at c . 5

(b) Use the ϵ - δ definition of the limit to prove that

$$\lim_{x \rightarrow c} x^3 = c^3 \text{ for any } c \in \mathbb{R}. \quad 5$$

(c) State divergence criterion for limit of a function. Show

that $\lim_{x \rightarrow 0} (x + \operatorname{sgn}(x))$ does not exist. 5

(d) Prove that :

$$(i) \quad \lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$$

$$(ii) \quad \lim_{x \rightarrow \infty} \frac{1}{x^2} = 0. \quad 5$$

(a) Let $A \subseteq \mathbb{R}$, $f, g, h : A \rightarrow \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A . If $f(x) \leq g(x) \leq h(x)$ for all $x \in A$, $x \neq c$ and if $\lim_{x \rightarrow c} f(x) = L = \lim_{x \rightarrow c} h(x)$, then prove that

$$\lim_{x \rightarrow c} g(x) = L. \quad 5$$

(b) State and prove sequential criterion for continuity of a real valued function. 5

(c) Let the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 2x & : \text{if } x \text{ is rational} \\ x+3 & : \text{if } x \text{ is irrational} \end{cases}$$

Find all the points at which f is continuous. 5

(d) Let $x \rightarrow [x]$ denote the greatest integer function. Determine the points of continuity of the function $f(x) = x - [x]$,

$x \in \mathbb{R}$. 5

3. (a) Let f be a continuous real valued function defined on $[a, b]$. By assuming that f is a bounded function show that f attains its bounds on $[a, b]$. 5
- (b) State Bolzano's Intermediate value theorem and show that the function $f(x) = xe^x - 2$ has a root c in the interval $[0, 1]$. 5
- (c) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous on \mathbb{R} and suppose that $f(r) = 0$ for every rational numbers r . Show that $f(x) = 0$ for all $x \in \mathbb{R}$. 5
- (d) Define uniform continuity of a function. Prove that if a function is continuous on a closed and bounded interval I , then it is uniformly continuous on I . 5
4. (a) Show that the function $f(x) = 1/x^2$ is uniformly continuous on $A = [0, \infty[$ but it is not uniformly continuous on $B =]0, \infty[$. 5
- (b) Determine where the following function $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable, $f(x) = |x - 1| + |x + 1|$. 5



- (c) Let f be defined on an interval I containing the point c . Then prove that f is differentiable at c if and only if there exists a function ϕ on I that is continuous at c and satisfies $f(x) - f(c) = \phi(c)(x - c)$ for all $x \in I$. In this case, we have $\phi(c) = f'(c)$. Using the above result find the function ϕ for $f(x) = x^3, x \in \mathbb{R}$ 5
- (d) State and prove Mean Value Theorem. 5
5. (a) State Darboux's theorem. Suppose that $f: [0, 2] \rightarrow \mathbb{R}$ is continuous on $[0, 2]$ and differentiable on $]0, 2[$ and that $f(0) = 0, f(1) = 1, f(2) = 1$. (i) Show that there exists $c_1 \in (0, 1)$ such that $f'(c_1) = 1$. (ii) Show that there exists $c_2 \in (1, 2)$ such that $f'(c_2) = 0$. (iii) Show that there exists $c \in (0, 2)$ such that $f'(c) = 1/10$. 5
- (b) Let $f: I \rightarrow \mathbb{R}$ be differentiable on the interval I . Then prove that f is increasing on I if and only if $f'(x) \geq 0$ for all $x \in I$. 5
- (c) State Taylor's theorem. Use it to prove that $1 - x^2/2 \leq \cos x$ for all $x \in \mathbb{R}$. 5
- (d) Find the Taylor series for e^x and state why it converges to e^x for all $x \in \mathbb{R}$. 5

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