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Roll No. ....

S. No. of Question Paper: 1852

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Unique Paper Code : **2351303**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics** ~~**Under Erstwhile FYUP**~~

Semester : **III**

Duration: **3 Hours**

Maximum Marks: 75

(Write your Roll No. on the top immediately on receipt of this question paper)

All questions are compulsory

Attempt any two parts from each question

Marks are indicated against each question

Use of Scientific Calculator is allowed

1. (a) Perform four iterations of the Newton Raphson method to obtain a root of $f(x) = x^3 - 5x + 1$ taking $p_0 = 0.5$.

(b) Define the order of convergence of an iterative method $\{x_n\}$. Determine the order of convergence of the recursive scheme $x_{n+1} = \frac{1}{2}(x_n + b/x_n)$.

(c) Describe the Bisection method to approximate the root of an equation. Perform three iterations of this method to approximate a zero of $f(x) = e^{-x} - x$ on the interval [0,1]. 13

2. (a) Derive the rate of convergence of the Regula Falsi method.

(b) Perform three iterations of the Regula Falsi method to determine a positive root of $f(x) = x^5 + 2x - 1$ in the interval [0,1].

(c) Find the approximate root of $f(x) = x^3 + 2x^2 - 3x - 1$ by secant method, taking

$p_0 = 2$ and $p_1 = 1$, until the error $|p_n - p_{n-1}| < 5 \times 10^{-3}$. 12

3. (a) Find an LU decomposition of the matrix:

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

and use it to solve the system $AX = [4 \quad 4 \quad 6]^T$.

(b) Perform three iteration of Gauss-Seidel method to solve the system of linear equations $AX = b$, where

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -3 & 1 \\ 0 & 2 & -3 \end{bmatrix}, b = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$$

Take $\boldsymbol{X}^{(0)} = \boldsymbol{0}$ as the initial approximation.

(c) Perform three iterations of Jacobi method to solve the system of linear equations

$$10x + 4y - 2z = 12$$

$$x - 10y - z = -10$$

$$5x + 2y - 10z = -3$$

Take $\boldsymbol{X}^{(0)} = [\boldsymbol{1} \quad \boldsymbol{1} \quad \boldsymbol{1}]$ as the initial approximation. 13

4. (a) For the following data

X	-1	2	4	5
f(x)	-5	13	255	625

obtain the Lagrange interpolating polynomial. Estimate f(3).

(b) Calculate the nth divided difference of 1/x, based on the points $x_0, x_1, \ldots, x_n$.

(c) For the function f(x)=sin x, obtain the Lagrange linear interpolating polynomial in the interval [1,3]. Obtain approximate values of f(1.5) and f(2.5). 12

5. (a) Define the forward difference operator Δ, central difference operator δ and average operator μ. Prove that:

(i) $\delta = \Delta(1+\Delta)^{-1/2}$

(ii) $\mu = \left(1+\frac{\Delta}{2}\right)(1+\Delta)^{1/2}$

(b) For the following data, obtain the forward and the backward Newton difference polynomials and interpolate at x=0.25 and x=0.35.

x	1	1.5	2.0	2.5
f(x)	2.7183	4.4817	7.3891	12.1825

(d) If $x_0, x_1, \ldots, x_n$ are n+1 distinct points and f is defined at these n+1 points, then prove that interpolating polynomial P, of degree at most n, is unique.

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6. (a) Find approximate value of $\int_0^1 \frac{1}{1+x^3}\,dx$ using composite Trapezoidal rule with 2 equal sub-intervals.

(b) Find approximate value of $\int_0^1 tan^{-1}x\,dx$ using Simpson's rule. Verify that the theoretical error bound is satisfied.

(c) Apply Euler's method to approximate the solution of the initial value problem

$$\frac{dx}{dt} = t^2 - 2x^2 - 1, \quad 0 \le t \le 1, \quad x(0) = 1$$

over the interval [0,1] using four steps.

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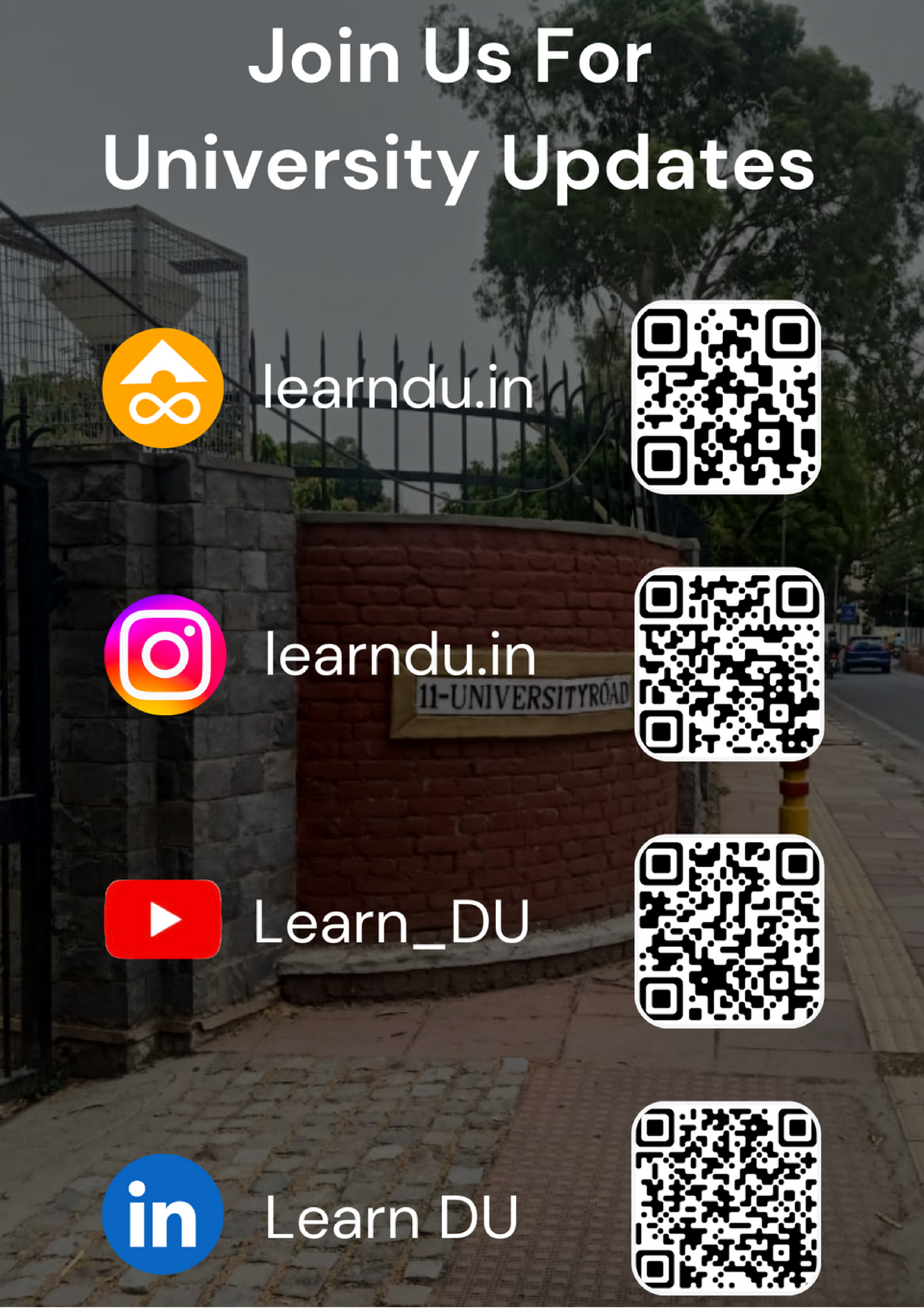
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