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Unique Paper Code : 235301

Name of the Paper : Calculus-II (MAHT-301)

Name of the Course : B.Sc.(H)- ~~II (Semester Scheme)~~ Mathematics

I

Semester : ~~3~~ (II)

Duration : 3 hours

Maximum Marks : 75

Instruction for the candidates

(Write your Roll No. on the top immediately on the receipt of this question paper)

All sections are compulsory.

Attempt any five questions from each section. All questions carry equal marks.

(Section – 1)

1. Define level curves and sketch the level curves of the function $f(x, y) = 10 - x^2 - y^2$ cut by the planes $z = 1, z = 3$ and $z = 7$.
2. Show that the limit of the function defined by $f(x, y) = \frac{x^2y}{x^4+y^2}$ does not exist at $(x, y) = (0,0)$.
3. Compute the slope of the tangent line to the graph of the function $f(x, y) = x^2 \sin(x + y)$ at the point $P_0(\frac{\pi}{2}, \frac{\pi}{2}, 0)$ in the direction parallel to xz-plane.
4. Use an incremental approximation to estimate the function $f(x, y) = e^{xy}$ at the point $(x, y) = (1.01, 0.98)$.
5. Given the function $F(x, y) = x^2 + y^2$, where $x = u \sin v$ and $y = u - 2v$. Find the partial derivatives $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ in two ways by considering $z = F(x(u, v), y(u, v))$.
6. Find all the relative extrema and saddle points of the function

$$f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

(Section – 2)

7. Evaluate the iterated integral $\int_1^2 \int_0^{\pi} x \cos y \, dy \, dx$.

8. Find the volume of a tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the coordinate planes.

9. Using cylindrical coordinates, find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half-cone $z = \sqrt{x^2 + y^2}$, and $xy-$ the plane.

10. Evaluate the triple integral $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} \int_0^{\cos\varphi} \rho^2 \sin\varphi \; d\rho \; d\theta \; d\varphi$.

11. If $u = xy$ and $v = x^2 - y^2$ express the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ in terms of u and v .

12. Evaluate $\int_0^2 \int_y^{\sqrt{(8-y^2)}} \frac{dx\,dy}{\sqrt{(1+x^2+y^2)}}$ using polar coordinates.

(Section – 3)

13. Evaluate the line integral $\oint_C x^2 z \, ds$, where C is the helix $x = \cos t, y = 2t, z = \sin t$, for $0 \le t \le \pi$.

14. a) Find the divergence of the function $\boldsymbol{F} = xy\,\boldsymbol{i} + yz\,\boldsymbol{j} + xz\,\boldsymbol{k}$. curl

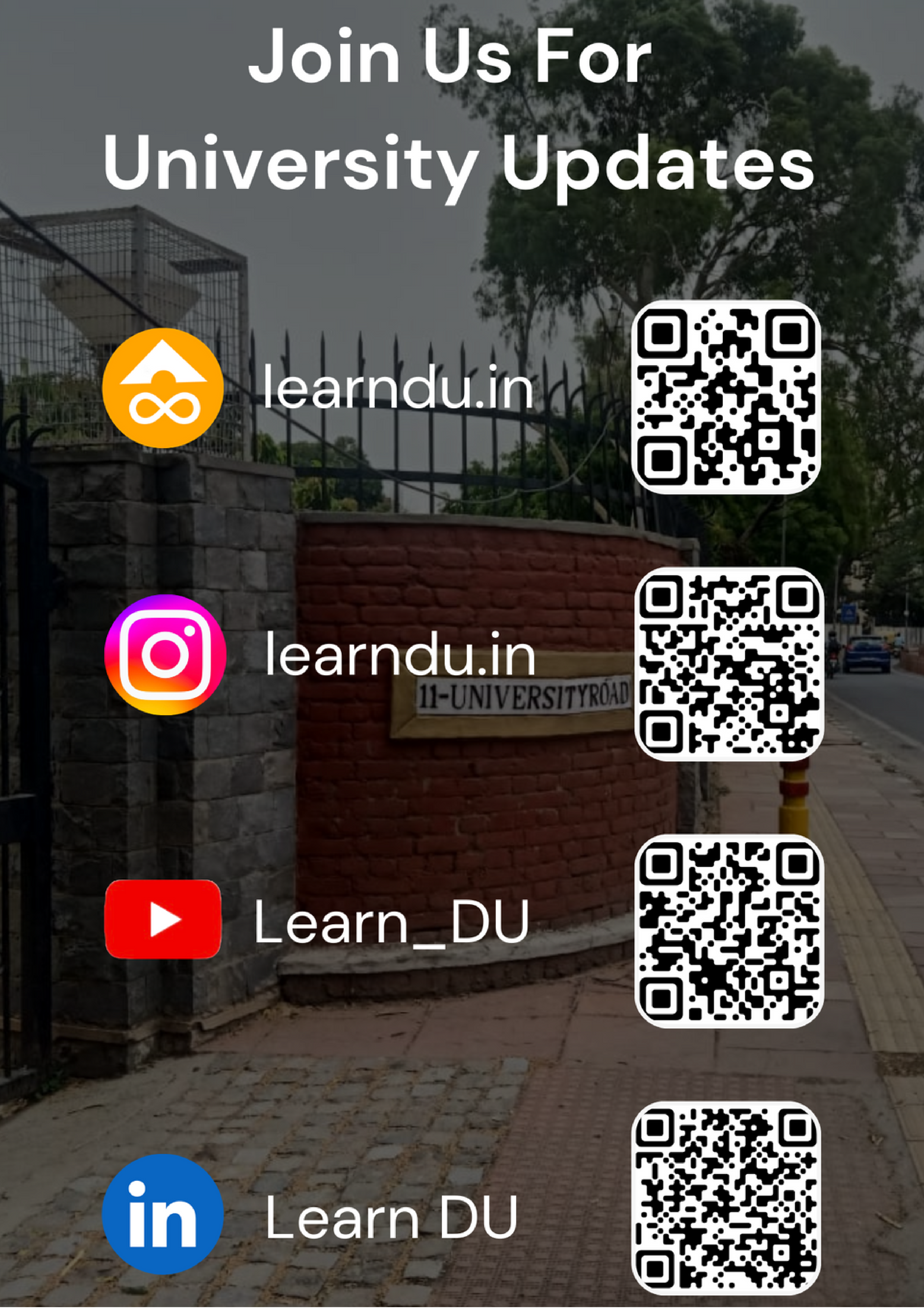
b) Let $\boldsymbol{R} = (x, y, z)$ and $r = \|\boldsymbol{R}\| = \sqrt{x^2 + y^2 + z^2}$. Show that $url\left(\frac{1}{r^3}\,\boldsymbol{R}\right) = 0$.

15. Find the work done by an object moving along the C in the force field $\boldsymbol{F}(x,y) = (x + xy^2)\hat{\imath} + 2(x^2y - y^2 \sin y)\hat{\jmath}$, where C is the closed path in the plane defined by the curve $y = x^2$ from $(0,0)$ to $(1,1)$, followed by the lines $y = 1$ and $x = 0$ from $(1,1)$ to $(0,1)$ and from $(0,1)$ to $(0,0)$ respectively.

16. Compute the flux integral $\iint_S \boldsymbol{F}.\boldsymbol{N}\,dS$ where $\boldsymbol{F} = xy\,\boldsymbol{i} + z\,\boldsymbol{j} + (x + y)\boldsymbol{k}$ and S is the triangular surface cut off from the plane $x + y + z = 1$ by the coordinate planes. Assume $\boldsymbol{N}$ is the upward unit normal.

17. Use divergence theorem to evaluate the surface integral $\iint_S \boldsymbol{F}.\boldsymbol{N}\,dS$ where $\boldsymbol{F} = x\,\mathrm{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and S is the closed bounded surface $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$, and assume $\boldsymbol{N}$ is the outward unit normal vector.

18. Let S be the portion of the plane $x + y + z = 1$ that lies in the first octant, and let C be the boundary of S , traversed counterclockwise as viewed from the above. Use Stoke's theorem to evaluate $\oint_C \boldsymbol{F}.d\boldsymbol{R}$ where $\boldsymbol{F} = -\frac{3}{2}y^2\,\boldsymbol{i} - 2xy\,\boldsymbol{j} + yz\,\boldsymbol{k}$.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