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2018

This question paper contains 2 printed pages

Roll No.

Unique paper code : 235304

Name of the course : B. Sc. (Hons) Mathematics

Name of the paper : MAHT 303 – Algebra II

Semester : III

Duration : 3 Hours

Maximum marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of the question paper.
2. Attempt any ***two*** parts from each question.
3. All questions are compulsory.

1. (a) Let H be a finite nonempty subset of a group G. Then, prove that H is a subgroup of G if and only if H is closed under the binary operation of G.

(b) Suppose that $|a| = 24$. Find a generator for $< a^{21} > \cap < a^{10} >$. In general, find a generator for the subgroup $< a^m > \cap < a^n >$.

(c) Let G be a finite cyclic group of order n. Prove that for every divisor d of n, there exists a unique subgroup of order n.

(6, 4+2, 6)

2. (a) Prove that every group of order 3 is Abelian.

(b) In a group G, prove that, for all $a, b \in G$, the equations $ax = b$ and $ya = b$ have unique solutions in G.

(c) In a G group, let $a, b \in G$ such that $(ab)^2 = a^2b^2$. Then, prove that $ab = ba$.

(6,6,6)

3. (a) Prove that the disjoint cycles commute.

(b) Define a cyclic group. Prove that every group of order p, where p is a prime, is cyclic.

(c) Prove that the set A_n of even permutations of the group S_n is a normal subgroup of S_n and $|A_n| = |S_n|/2$, where $n \geq 3$.

(6,6,6)

4. (a) State and prove Lagrange's Theorem.

(b) Define a coset of a subgroup and a normal subgroup of a group G. Prove that a subgroup H of G is normal if and only if $xHx^{-1} \subseteq H$.

(c)(i) Let H be a subgroup of a group G having the index 2. Then, prove that H is a normal subgroup of G.

(ii) Let H be a subgroup of a group G such that $x^2 \in H$, for all $x \in G$. Then, prove that H is a normal subgroup of G.

($2+4\frac{1}{2}, 2+4\frac{1}{2}, 4+2\frac{1}{2}$)

5. (a) Let G be a group. Then, prove that $\frac{G}{Z(G)} \cong Inn(G)$.

(b) (i) Find $Aut(\mathbb{Z})$.

(ii) Let G be a group and φ be a mapping from G to G defined by $\varphi(g) = g^{-1}$. Then, prove that φ is an automorphism if and only if G is Abelian.

(c) Let φ be a homorphism from a group G to a group $\bar{G}$ and let $g \in G$. If $\varphi(g) = \bar{g}$, then prove that $\varphi^{-1}(\bar{g}) = g \ker \varphi$.

($6\frac{1}{2}, 3+3\frac{1}{2}, 6\frac{1}{2}$)

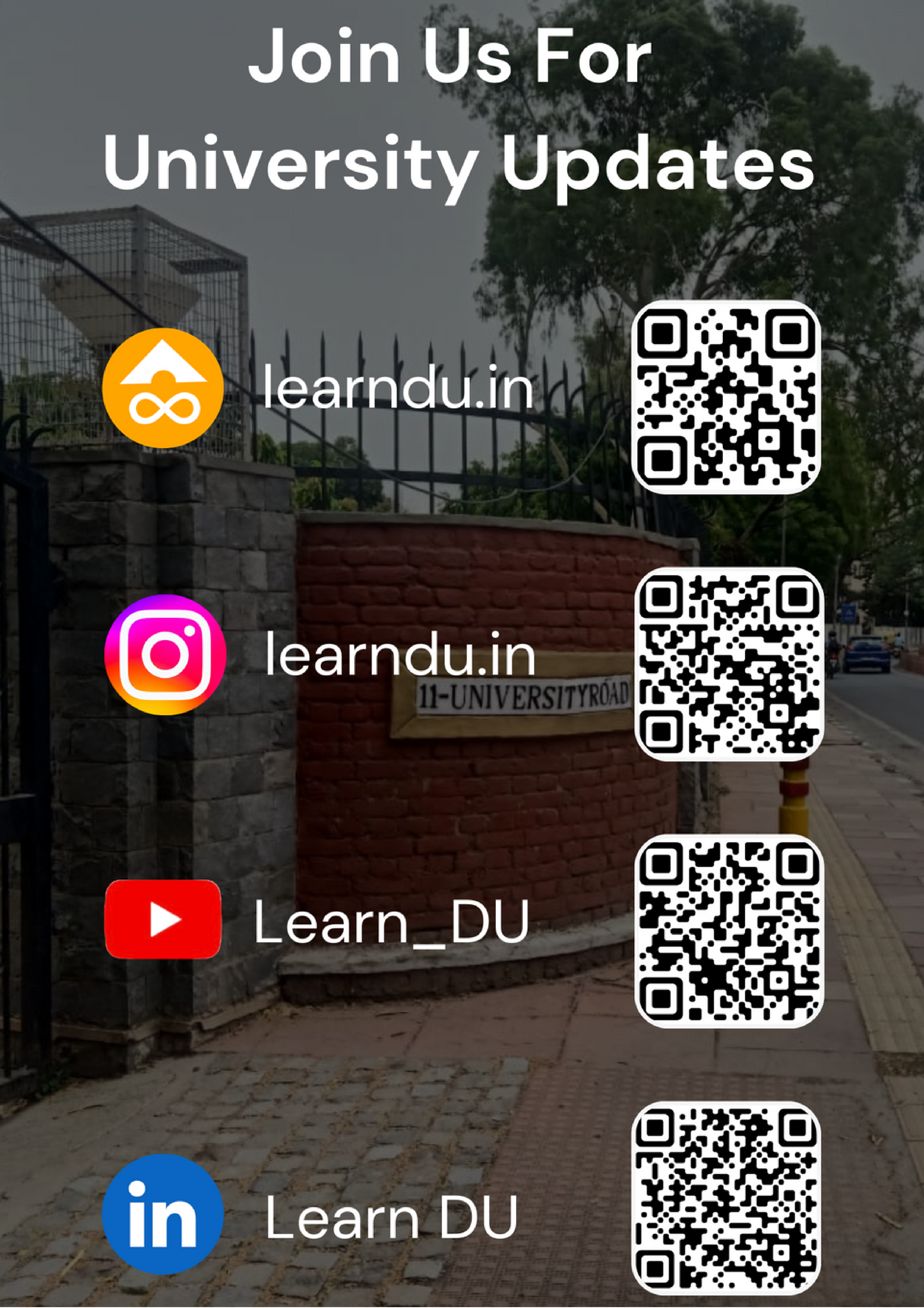
6. (a) Prove that every group is isomorphic to a group of permutations.

(b) Prove that any finite cyclic group of order n is isomorphic to Z_n and any infinite cyclic group is isomorphic to $\mathbb{Z}$.

(c) Let φ be a homorphism from a group G onto a group $\bar{G}$. Then, prove that

$$\frac{G}{\ker \varphi} \cong \bar{G}.$$

($6\frac{1}{2}, 6\frac{1}{2}, 6\frac{1}{2}$)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