Learn DU

MAKE IT BIG!

All The Best

For Your Exams

[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 8897   14   Your Roll No. 2019

Unique Paper Code   : 235104

Paper Code   : MAHT 103

Name of the Course   : B.Sc. (Hons.) Mathematics – I

Name of the Paper   : Algebra-I

Semester   : 1

Duration: 3 Hours   Maximum Marks:75

Instruction for Candidates

1) All six questions are compulsory.

2) Do any two parts from each question.

3) Marks for each part of a question are written against the question in the margin.

1. a) Find the polar representation of complex number   6

$$z = \cos a - i \sin a , a \in [0, 2\pi)$$

   b) Compute the following   6

$$z^n + \frac{1}{z^n}, \text{ if } z + \frac{1}{z} = \sqrt{3} .$$

   c) Find the quadratic equation whose roots are the cubes of the roots of the   6
      equation $x^2 - px + q = 0$.

2. a) For $a, b \in \mathbb{R} \setminus \{0\}$, define $a \sim b$ if and only if $\frac{a}{b} \in \mathbb{Q}$ 6

      i.   Prove that $\sim$ defines an equivalence relation .

      ii. What is an equivalence class of 1? Show that $\overline{\sqrt{3}} = \overline{\sqrt{12}}$.

   b) Given three consecutive integers $a, a + 1, a + 2$, prove that one of them is   6
      divisible by 3.

   c)   ~~a)~~ Define $f: \mathbb{Z} \to \mathbb{Z}$ by   6

$$f(x) = 3x^3 + x .$$

      Determine whether or not f is one to one and/or onto .

3. a) Use mathematical induction to establish that for all $n \geq 1$,   6
       $8^n - 3^n$ is divisible by 3.

    b) Show that the set of rational numbers is countable.   6

c) Find all integers x, $0 \le x < 6$, satisfying the following congruence $\hspace{1cm}$ 6

$\qquad$ $4x \equiv 2 \pmod 6$.

4. $\quad$ a) $\;$ Find the general solution of the system $\hspace{4cm}$ $6\frac{1}{2}$

$$2x_1 - x_2 + x_3 + 2x_4 = 0$$
$$-2x_1 + 4x_2 - x_3 + 2x_4 = -5$$
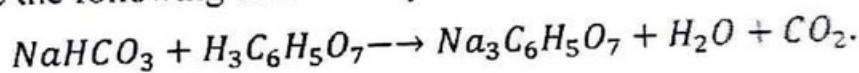$$x_1 - 6x_2 + 3x_3 + x_4 = 7$$
$$4x_1 - 6x_2 + x_3 - 4x_4 = 9$$

by reducing the coefficient matrix to echelon form.

b) $\;$ Determine whether b belongs to the linear span of a_1, a_2 and a_3, where $\hspace{1cm}$ $6\frac{1}{2}$

$$a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, a_2 = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}, a_3 = \begin{pmatrix} -6 \\ 7 \\ 5 \end{pmatrix}, \text{ and } b = \begin{pmatrix} 11 \\ -5 \\ 9 \end{pmatrix}.$$

c) $\;$ Balance the following chemical equation $\hspace{5cm}$ $6\frac{1}{2}$

$$NaHCO_3 + H_3C_6H_5O_7 \longrightarrow Na_3C_6H_5O_7 + H_2O + CO_2.$$

5. $\quad$ a) $\;$ For what values of h the vectors v_1, v_2 and v_3 given below $\hspace{2cm}$ $6\frac{1}{2}$

$$v_1 = \begin{pmatrix} 3 \\ -6 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}, v_3 = \begin{pmatrix} 9 \\ h \\ 3 \end{pmatrix},$$

are linearly dependent?

b) $\;$ Let $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, y_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, and $y_2 = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be such $\hspace{1cm}$ $6\frac{1}{2}$

$\qquad$ that $Te_1 = y_1$ and $Te_2 = y_2$. Find $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

c) $\;$ (i) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Show that T is one-to-one if $\hspace{1cm}$ $6\frac{1}{2}$

$\qquad$ and only if $T(x) = 0$ has only the trivial solution.

$\qquad$ (ii) Show that $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x_1, x_2) = (x_1 + x_2, x_2)$ is one-to-one.

6. $\quad$ a) $\;$ Find the standard matrix of the horizontal sheer transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ that $\hspace{0.5cm}$ $6\frac{1}{2}$

$\qquad$ leaves e_1 unchanged and maps e_2 into $e_2 + 2e_1$.

b) $\;$ Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transform and A be standard matrix representation of $\hspace{0.2cm}$ $6\frac{1}{2}$

$\qquad$ T. Show that T is invertible linear transformation if and only if A is an invertible

$\qquad$ matrix.

$\hspace{14cm}$ $6\frac{1}{2}$

c) $\;$ Determine the rank of the matrix

$$\begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{pmatrix}.$$

Join Us For University Updates

learndu.in

learndu.in

Learn_DU

Learn DU